Introduction to Mathematics and Modeling

lecture 8

Integration by parts

UNIVERSITY OF TWENTE.

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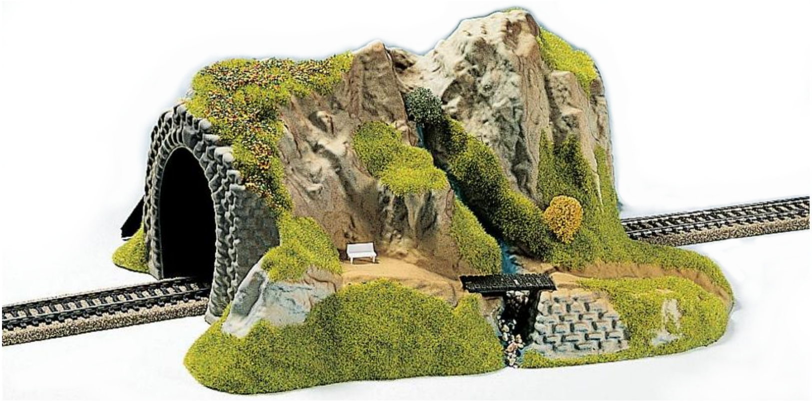
Part 2



1. Section 8.1: integration by parts
2. Section 8.2: trigonometric integrals

$$\boxed{\int f'(x)g(x)\ \mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\ \mathrm{d}x}$$

Proof:

- Recall the product rule for differentiation:
$$\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x).$$
- Integrating both sides gives
$$\begin{aligned}\int \frac{d}{dx}f(x)g(x)\ \mathrm{d}x &= \int f'(x)g(x) + f(x)g'(x)\ \mathrm{d}x \\ f(x)g(x) &= \int f'(x)g(x)\ \mathrm{d}x + \int f(x)g'(x)\ \mathrm{d}x.\end{aligned}$$
- Moving the second term to the left gives the boxed formula.

For definite integrals the rule reads as

$$\begin{aligned}\int_a^b f'(x)g(x)\ \mathrm{d}x &= [f(x)g(x)]_a^b - \int_a^b f(x)g'(x)\ \mathrm{d}x \\ &= f(b)g(b) - f(a)g(a) - \int_a^b f(x)g'(x)\ \mathrm{d}x.\end{aligned}$$

Example 1, alternative 1

$$\int xe^{x}\,\mathrm{d}x = ??$$

$$\int xe^x \,\mathrm{d}x = ??$$

$$\boxed{\int f'(x)g(x)\ \mathrm{d}x = f(x)g(x) - \int f(x)g'(x)\ \mathrm{d}x}$$

Notate $f(x) = u$ and $g(x) = v$, then the rule becomes

$$\boxed{\int u'v\ \mathrm{d}x = uv - \int u\,v'\ \mathrm{d}x}$$

$$\int xe^x \, \mathrm{d}x = ??$$

$$\int x^2 e^{-x}\,\mathrm{d}x = ??$$

$$\int x \ln(x)\ \mathrm{d}x = ??$$

Example 4

$$\int \ln(x)\ \mathrm{d}x = ??$$

- With the explicit renaming $f(x) = u$ and $g(x) = v$:

$$\boxed{\int u'v \ \mathrm{d}x = uv - \int u\, v' \ \mathrm{d}x}$$

- Note that $du = u' \, \mathrm{d}x$ and $dv = v' \, \mathrm{d}x$. Therefore the rule can be memorized as follows:

$$\boxed{\int v \, \mathrm{d}\, u = uv - \int u \, \mathrm{d}\, v}$$

- You can even do this *without* renaming f and g:

$$\boxed{\int g(x) \ \mathrm{d}f(x) = f(x)g(x) - \int f(x) \ \mathrm{d}\, g(x)}$$

- If $u = g(x)$ then

$$du = g'(x)\,dx.$$

- Write

$$d(g(x)) = g'(x)\,dx.$$

- From right to left: *differentiate*, from left to right: *integrate*:

$$d(x^2 + 1) \qquad d(\tfrac{1}{3}x^3) \qquad d(e^{2x})$$

$$2x\,dx \qquad x^2\,dx \qquad 2e^{2x}\,dx$$

- You may add an arbitrary constant to the right hand side:

$$2x\,dx = d\,x^2 = d\,(x^2 + 36).$$

$$\int (2x+1)e^x \, \mathrm{d}x = ??$$

$$\int x^3 e^{x^2}\,\mathrm{d}x = ??$$

Example 6, alternative 1

$$\int x^3 e^{x^2} \, \mathrm{d}x = ??$$

Example 6, alternative 2

$$\int x^3 e^{x^2}\ \mathrm{d}x = ??$$

Example 7

$$\int e^{\sqrt{x}}\,\mathrm{d}x = ??$$

$$I = \int e^x \cos(x) \, \mathrm{d}\, x = ??$$

$$\int \cos(\ln x)\ \mathrm{d}\,x = ??$$

Let m and n be non-negative integers.

$$\int \sin^m x \cos^n x \, \mathrm{d}x = ??$$

The following formulas are useful:

- $\sin^2 x + \cos^2 x = 1$
- $\sin x \cos x = \frac{1}{2} \sin(2x)$
- $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
- $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

Example 10, alternative 1

$$\int \sin^2 x \cos^2 x \, \mathrm{d}x = ??$$

Example 10, alternative 2

$$\int \sin^2 x \cos^2 x \ \mathrm{d}x = ??$$

$$\int \cos^4 x \ \mathrm{d}x = ??$$

Example 12

$$\int \cos x \sin^2 x \; \mathrm{d}x = ??$$

$$\int \cos x \, \sin(2x) \, \mathrm{d}x = ??$$